CAPM WITH VARIOUS RISK MEASURES: LOW-BETA ANOMALY IN THE U.S. AND EUROPEAN EQUITIES

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Aim of study

- use new dynamic portfolio performance risk measures of standard, downside, CVaR, ERoD and CDaR Beta to investigate the CAPM linear relationship between the expected return of an asset and its systematic risk;
- study the low beta anomaly: low-beta stocks outperform high-beta stocks in the U.S. and European stock markets.

Data and sample description

- Daily closing prices: 1995-01-01 – 2024-12-31.
- Stocks: S&P500 and STOXX Europe 600 companies listed in U.S. dollars and euros or in domestic currencies.
- Three 10 years periods:
  Period 1: 1995-01-01 – 2004-12-31
  Period 2: 2005-01-01 – 2014-12-31
  Period 3: 2015-01-01 – 2024-12-31

Research methodology

The Generalized Capital Asset Pricing Model (GCAPM) is an extension of the classical CAPM that allows for more realistic assumptions about risk and investor behavior. Different deviation measures result in corresponding generalized beta coefficients, which reflect the marginal contribution of each asset to the overall portfolio risk. Considered cases (standard, Downside, CVaR, CDaR, ERoD):

$$(1)\ \beta^i_{Standard} = \frac{Cov\left(r^i, r^M\right)}{Var\left(r^M\right)}$$

$$(2)\beta^i_{Downside} = \frac{Cov\left(r^i, r^M | r^M < \bar{r}^M\right)}{Var\left(r^M | r^M < \bar{r}^M\right)}$$

$$(3)\ \beta^i_{CVaR} = \frac{\lambda_1 E\left[Er^i - r^i \,|r_M \le -VaR_{\alpha_1}\left(r^M\right)\right] + \cdots \lambda_L E\left[Er^i - r^i \,|r_M \le -VaR_{\alpha_L}\left(r^M\right)\right]}{\lambda_1 CVaR^{\Delta}_{\alpha_1}\left(r^M\right) + \cdots + \lambda_L CVaR^{\Delta}_{\alpha_L}\left(r^M\right)}$$

$$(4)\ \beta^i_{CDaR,\alpha} = \frac{\sum_{t=1}^{T} q_t\left(w^i_{\tau(t)} - w^i_t\right)}{\sum_{t=1}^{T} q_t\left(w^M_{\tau(t)} - w^M_t\right)}$$

$$(5)\ \beta^i_{ERoD,\epsilon} = \frac{\frac{1}{T}\sum_{t=1}^{T} q^*_t\left(w^i_{\tau(t)} - w^i_t\right)}{\frac{1}{T}\sum_{t=1}^{T} q^*_t\left(w^M_{\tau(t)} - w^M_t\right)}$$

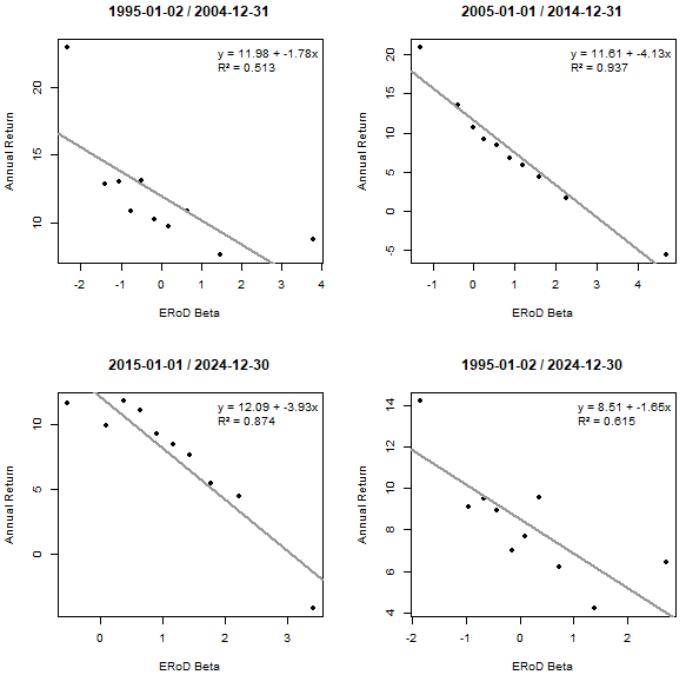
Steps for each beta parameter specification:
1. In a given period, we divided companies into ten equal groups based on the quartiles of the distribution of a given beta parameter specification.
2. For each group, we determined the mean value of the given beta parameter specification and the annualized returns.
3. We performed a regression of the mean annualized returns on the mean beta parameter values.

Results

Table 1. Summary of regression results for U.S. market.

Period:	$\hat{\alpha}$	$\hat{\beta}$	$t_{\hat{\alpha}}$	$t_{\hat{\beta}}$	R^2
ERoD: 1995/2004	11.975	-1.781	11.991 ***	-2.901 **	0.513
ERoD: 2005/2014	11.606	-4.134	16.608 ***	-10.918 ***	0.937
ERoD: 2015/2024	12.093	-3.931	14.499 ***	-7.439 ***	0.874
ERoD: 1995/2024	8.508	-1.646	15.073 ***	-3.577 ***	0.615
CDaR: 1995/2004	11.423	-2.141	24.717 ***	-7.388 ***	0.872
CDaR: 2005/2014	16.081	-9.046	19.024 ***	-11.505 ***	0.943
CDaR: 2015/2024	7.912	-0.455	6.946 ***	-0.499	0.030
CDaR: 1995/2024	8.523	-1.628	31.137 ***	-5.702 ***	0.803
Standard: 1995/2004	11.004	1.080	9.268 ***	0.969	0.105
Standard: 2005/2014	14.393	-5.371	11.389 ***	-5.92 ***	0.814
Standard: 2015/2024	10.860	-2.889	4.591 ***	-1.5	0.219
Standard: 1995/2024	11.260	-2.705	5.587 ***	-1.559	0.233
Downside: 1995/2004	10.547	1.389	11.581 ***	1.949 *	0.322
Downside: 2005/2014	12.796	-3.857	19.613 ***	-9.127 ***	0.912
Downside: 2015/2024	13.137	-4.391	7.911 ***	-3.799 ***	0.643
Downside: 1995/2024	12.441	-3.372	8.63 ***	-3.127 **	0.550
CVaR: 1995/2004	7.766	4.934	5.733 ***	3.41 ***	0.592
CVaR: 2005/2014	16.365	-7.603	6.194 ***	-3.479 ***	0.602
CVaR: 2015/2024	6.920	0.614	3.808 ***	0.365	0.016
CVaR: 1995/2024	10.189	-1.869	4.5 ***	-0.852	0.083

Fig 1. Regression of annual return on ERoD Beta (U.S. market).



Conclusions

- Among considered measures, the ERoD beta consistently delivers the most stable performance in both markets. CDaR-based betas also demonstrate strong predictive capacity, especially in earlier periods, although their performance weakens somewhat in recent years.
- The classical standard deviation beta exhibits much weaker and less consistent performance in the U.S. market but shows more strength in Europe, especially in more recent decades.
- The low-beta anomaly is highly visible in both markets regarding different deviation measures, but stronger for the European stocks than the American ones. It shows that low-beta stocks often earn similar or even higher returns than high-beta stocks (which indicates that the effective borrowing rate is higher than the expected market return). The anomaly is persistent across markets and three subperiods globally not just in the U.S.